

New gauge vector in lyra geometry and application in cosmology

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Abstract

Many theorists have studied cosmology within Lyra geometry. However, the field equations derived from the gauge vector they assumed are not exactly the same as the well-known cosmological equations derived from Riemannian geometry. In our paper, we introduced a new gauge vector to overcome this difficulty. The field equations derived from our gauge is exactly the same as the well-known cosmological equations.

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1 Introduction

Einstein has founded his magnificent theory—General Relativity, which is based on Riemannian geometry. With modern points of view, the dark energy has a dominant portion in space, but in Einstein's regime, the cosmological constant Λ , just plunged into the Einstein equation with out any intrinsic geometry implication as Sen pointed out. Weyl[1] has proposed a modified Riemannian geometry to formulate a unified field theory but failed for physical reasons. Later Lyra[2] proposed a new modification of Riemannian geometry upon the work of Sen. The cosmological constant arises naturally from Lyra's geometry. To be precise, it relates to a gauge vector $(\beta, \alpha_1, \alpha_2, \alpha_3)$.

Many theorists have studied cosmology by using Lyra geometry[3-24] and got a lot of good results. They all assumed that the gauge vector is only have a time component, which is of the form $(\beta, 0, 0, 0)$. However, the field equations derived from this gauge is not exactly the same as the well-known cosmological equations. Now we assume the spatial component of the gauge vector is not zero. After a series of calculation, we get a new gauge vector. The field equations derived from it is just exactly the same as the well-known cosmological equations. Further more, the time component behaves just like the previous research assumed.

2 The Lyra geometry and field equations

First of all, we rewrite the Einstein field equation without the cosmological constant:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = -\kappa GT_{\mu\nu} \quad (1)$$

The LHS is the Einstein Tensor. The RHS is the energy-momentum tensor, which is:

$$T_0^0 = \rho, T_1^1 = T_2^2 = T_3^3 = -p, T_\nu^\mu = 0 (\mu \neq \nu) \quad (2)$$

If we add the cosmological constant Λ in the equation above, we'll get:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = -\kappa GT_{\mu\nu} \quad (3)$$

We can write down the equation in the components.

$$G_{ii} + \Lambda g_{ii} = -\kappa T_{ii} \quad 1 \leq i \leq 3 \quad (4)$$

$$G_{00} + \Lambda g_{00} = -\kappa T_{00} \quad (5)$$

Secondly, because of our universe is isotropic and homogeneous, which is the cosmological principle. This spacetime symmetry is concluded as the FRW model. The line element is

$$ds^2 = dt^2 - S^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right] \quad (6)$$

We put the metric into the Einstein tensor, we could get the representation of the components:

$$-G_1^1 = -G_2^2 = -G_3^3 = \frac{k}{S^2} + \frac{\dot{S}^2}{S^2} + \frac{2\ddot{S}}{S} \quad (7)$$

$$-G_0^0 = \frac{3k}{S^2} + \frac{3\dot{S}^2}{S^2} \quad (8)$$

Within the regime of Lyra geometry, Follow the standardized method[25] we get the field equation in lyra geometry[10]:

$$G_\nu^\mu + \frac{3}{2}\phi^\mu\phi_\nu - \frac{3}{4}\delta_\nu^\mu\phi^\alpha\phi_\alpha = -\kappa T_\nu^\mu \quad (9)$$

In the equation, ϕ_μ is the gauge vector in Lyra geometry.

3 Ordinary gauge vector model

In the is case, we just assume $\phi^\mu = (\beta, 0, 0, 0)$. Put it into the equation (9), we get the equations explicitly[10]:

$$\frac{k}{S^2} + \frac{\dot{S}^2}{S^2} + \frac{2\ddot{S}}{S} - \frac{3}{4}\beta^2 = -\kappa p \quad (10)$$

and

$$\frac{3k}{S^2} + \frac{3\dot{S}^2}{S^2} + \frac{3}{4}\beta^2 = \kappa \rho \quad (11)$$

Compared with the normal relativistic cosmology equations:

$$\frac{k}{S^2} + \frac{\dot{S}^2}{S^2} + \frac{2\ddot{S}}{S} - \Lambda = -\kappa p \quad (12)$$

$$\frac{3k}{S^2} + \frac{3\dot{S}^2}{S^2} - \Lambda = \kappa \rho \quad (13)$$

We can see an obvious difference between them is that the sign of β^2 is opposite. The theorist just studied the equation (13) and got a lot of results.

4 New gauge vector model

Instead of the ordinary vector $(\beta, 0, 0, 0)$, we use a 4-vector with spatial component. We just postulate that the gauge vector is:

$$\phi^\mu = (\beta, \alpha_1, \alpha_2, \alpha_3) \quad (14)$$

In which α_i and β are functions of time and space. Immediately, we can derive that:

$$\phi^0 = \phi_0 = \beta \quad (15)$$

$$\phi_1 = g_{11}\phi^1 = \frac{-S^2(t)}{1 - kr^2}\alpha_1 \quad (16)$$

$$\phi_1\phi^1 = -\alpha_1^2 \frac{S^2(t)}{1 - kr^2} \quad (17)$$

Also, we can get

$$\phi_2\phi^2 = -\alpha_2^2 S^2(t)r^2 \quad (18)$$

$$\phi_3\phi^3 = -\alpha_3^2 S^2(t)r^2 \sin^2\theta \quad (19)$$

We just put (15),(17),(18),(19) into the field equation (9) derived from lyra geometry. Here comes out 4 equations corresponding the 4-spacetime dimensions:

$$G_1^1 + \frac{3}{2}\phi_1\phi^1 - \frac{3}{4}\phi^\alpha\phi_\alpha = -\kappa T_1^1 \quad (20)$$

$$G_2^2 + \frac{3}{2}\phi_2\phi^2 - \frac{3}{4}\phi^\alpha\phi_\alpha = -\kappa T_2^2 \quad (21)$$

$$G_3^3 + \frac{3}{2}\phi_3\phi^3 - \frac{3}{4}\phi^\alpha\phi_\alpha = -\kappa T_3^3 \quad (22)$$

$$G_0^0 + \frac{3}{2}\phi_0\phi^0 - \frac{3}{4}\phi^\alpha\phi_\alpha = -\kappa T_0^0 \quad (23)$$

We want the new gauge vector could induce the right field equation like the ones in GR, so we need the following conditions:

$$\phi_1\phi^1 = \phi_2\phi^2 = \phi_3\phi^3 \quad (24)$$

Then, the first 3 equations (20),(21),(22) are the same, we focus on (20),(23) and then we get the field equations in the explicit form:

$$\frac{k}{S^2} + \frac{\dot{S}^2}{S^2} + \frac{2\ddot{S}}{S} - \frac{3}{4}\phi_1\phi^1 - \frac{3}{4}\beta^2 = -\kappa p \quad (25)$$

and

$$\frac{3k}{S^2} + \frac{3\dot{S}^2}{S^2} - \frac{9}{4}\phi_1\phi^1 + \frac{3}{4}\beta^2 = \kappa\rho \quad (26)$$

Compared with the normal relativistic cosmology equations:

$$\frac{k}{S^2} + \frac{\dot{S}^2}{S^2} + \frac{2\ddot{S}}{S} - \Lambda = -\kappa p \quad (27)$$

$$\frac{3k}{S^2} + \frac{3\dot{S}^2}{S^2} - \Lambda = \kappa\rho \quad (28)$$

So if the condition $-\frac{9}{4}\phi_1\phi^1 + \frac{3}{4}\beta^2 = -\frac{3}{4}\phi_1\phi^1 - \frac{3}{4}\beta^2$ holds, the equations are identical. Now we have $-\frac{9}{4}\phi_1\phi^1 + \frac{3}{4}\beta^2 = -\frac{3}{4}\phi_1\phi^1 - \frac{3}{4}\beta^2$, which means $\phi_1\phi^1 = \beta^2$. We can simplify the equations we have got, we can rewrite down the cosmology equations

$$\frac{k}{S^2} + \frac{\dot{S}^2}{S^2} - \frac{3}{2}\beta^2 = \kappa\rho \quad (29)$$

$$\frac{3k}{S^2} + \frac{3\dot{S}^2}{S^2} - \frac{3}{2}\beta^2 = \kappa p \quad (30)$$

So we have $\frac{3}{2}\beta^2 = \Lambda$. The Λ is a positive real number. So, another outcome is that if we want the cosmology derived from Lyra manifold is exactly the same as the General Relativity situation, we need to get a gauge vector that has a time component β which is a real number to make lyra manifold to be consolidated.

We can calculate the gauge vector explicitly.

$$\beta = \sqrt{\frac{2}{3}\Lambda} \quad (31)$$

$$\alpha_1 = \frac{\sqrt{\frac{2}{3}\Lambda}(1 - kr^2)}{S(t)}i \quad (32)$$

For the α_1 component if we suppose that our universe is flat or open, then the α_1 is pure imaginary. If the k is a positive real number, the α_1 component will be either real or imaginary in different patches of our universe, which gives us a hint that the universe might not be closed.

$$\alpha_2 = \frac{\sqrt{\frac{2}{3}\Lambda}}{S(t)r}i \quad (33)$$

$$\alpha_3 = \frac{\sqrt{\frac{2}{3}\Lambda}}{S(t)r\sin\theta}i \quad (34)$$

The Lyra gauge vector consolidated is:

$$\phi^\mu = \frac{\sqrt{\frac{2}{3}\Lambda}}{S(t)}(S(t), \sqrt{1 - kr^2}i, \frac{1}{r}i, \frac{1}{r\sin\theta}i) \quad (35)$$

We have mentioned that this gauge vector has imaginary components part, just like Sen pointed in[3], this kind of gauge vector can exist in the geometry without contradiction.

5 vacuum Friedman equation

At last, we can get the vacuum Friedman equation from (29),

$$H^2 + \frac{k}{S^2} - \frac{\beta^2}{2} = 0. \quad (36)$$

Where H is Hubble parameter. (36) could be used to describe the de Sitter universe. Neglect the influence of matter, the vacuum is the dominate faction in the universe, which is also adaptable to the inflation stage of the evolution of the early universe.

The solutions[26] of (36) is

- $k = 0$ $S(t) = m\exp(\frac{\sqrt{\Lambda}}{3}t)$
- $k = +1$ $S(t) = \frac{3}{\sqrt{\Lambda}}\cosh(\frac{\sqrt{\Lambda}}{6}t)$
- $k = -1$ $S(t) = \frac{3}{\sqrt{\Lambda}}\cosh(\frac{\sqrt{\Lambda}}{6}t)$

The Λ above is precisely detected now[27]: $\Lambda \sim 10^{-83} \text{Gev}^2$.

6 Discussion

Due to our work, we got a new gauge vector which could make a perfect corresponding with the lyra manifold to the Cosmology derived from Riemannian Geometry. Also, we gave the dark energy a good geometry implication that the dark energy is due to the new 4-dimensional gauge vector. The spatial part of our gauge vector is pure imaginary in the situation that our universe is flat or open. We will get an ill defined christoffel. Even though, the result of this is tolerable because the Ricci Tensor and Ricci scalar is still real indeed. The curvature is the one corresponds to the real physics. Later we specially focused on the vacuum Friedman Equation to get a image about the evolution of the vacuum. With our model, we ruled out the possibility discussed by A.Beesham that the universe has the possibility to oscillate, which means that the oscillation situation could be avoided in lyra geometry. We could also get a hint that our universe may be flat or open, not closed from the gauge vector.

Acknowledgments

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7 Reference

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